FLAVOR SYMMETRY AND NEUTRINO OSCILLATIONS

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We show how the nearly bi-maximal mixing scenario comes out naturally from gauged $SO(3)_F$ flavor symmetry via spontaneous symmetry breaking. An interesting relation between the neutrino mass-squared differences and the mixing angle, i.e., $\Delta m^2_{\mu e}/\Delta m^2_{\tau\mu} \simeq 2|U_{e3}|^2$ is obtained. The smallness of the ratio (or $|U_{e3}|$) can also naturally be understood from an approximate permutation symmetry. Once the mixing element $|U_{e3}|$ is determined, such a relation will tell us which solution will be favored within this model. The model can also lead to interesting phenomena on lepton-flavor violations.

1 Introduction

The greatest success of the standard model (SM) is the gauge symmetry structure $SU(3)_c \times SU_L(2) \times U_Y(1)$ which has been tested by more and more precise experiments. In the SM, neutrinos are assumed to be massless. The recent evidences for oscillation of atmospheric neutrinos[1] and for the deficit of the measured solar neutrino flux[2] strongly suggest that neutrinos are massive though their masses are small, and new physics beyond the SM is necessary. The scenario most favoured by the current data[3] may comprise just three light neutrinos with nearly bimaximal mixing via MSW solution[4]. It is of interest to note that such a scenario was shown to be naturally obtained from a simple extention of the SM with gauged SO(3)$_F$ flavor symmetry[5]. In this talk I mainly describe the most interesting features resulting from such a simply extended model.

2 The model

For a less model-dependent analysis, we directly start from an $SO(3)_F \times SU(2)_L \times U(1)_Y$ invariant effective lagrangian with three $SO(3)_F$ Higgs triplets

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}g_3' A_\mu^k (\bar{L}_i\gamma^\mu (t^k)_{ij} L_j + \bar{e}_{Ri}\gamma^\mu (t^k)_{ij} e_{Rj}) \\ &+(Y_{1ij}\bar{L}_i\phi_1 e_{R\,j} + Y_{2ij}\bar{L}_i\phi_2\phi_2^T L_j^c + H.c.) \\ &+D_\mu\varphi^* D^\mu\varphi + D_\mu\varphi^{'*} D^\mu\varphi' + D_\mu\varphi^{''*} D^\mu\varphi'' \\ &-V_\varphi + \mathcal{L}_{SM} \qquad (1)\end{aligned}$$

with effective Yukawa couplings

$$\begin{aligned}Y_{1ij} &= c_1\varphi_i\varphi_j\chi + c_1'\varphi_i'\varphi_j'\chi' + c_1''\varphi_i''\varphi_j''\chi'' \\ Y_{2ij} &= c_0\varphi_i\varphi_j^* + c_0'\varphi_i'\varphi_j^{'*} + c_0''\varphi_i''\varphi_j^{''*} + c\delta_{ij}\end{aligned}$$

$\mathcal{L}_{SM}$ denotes the lagrangian of the standard model. $\bar{L}_i(x) = (\bar{\nu}_i, \bar{e}_i)_L$ (i=1,2,3) are the SU(2)$_L$ doublet leptons and $e_{R\,i}$ $(i = 1,2,3)$ are the three right-handed charged leptons. $A^i_\mu(x)t^i$ $(i = 1,2,3)$ are the $SO(3)_F$ gauge bosons with t^i the $SO(3)_F$ generators and g_3' is the corresponding gauge coupling constant. Here $\phi_1(x)$ and $\phi_2(x)$ are two Higgs doublets, $\varphi(x)$, $\varphi'(x)$ and $\varphi''(x)$ are three $SO(3)_F$ Higgs triplets, and $\chi(x)$, $\chi'(x)$ and $\chi''(x)$ are three singlet scalars. The couplings c, c_a, c_a' and c_a'' $(a = 0,1)$ are dimensional constants. The structure of the above effective lagrangian can be obtained by imposing an additional U(1) symmetry [5].

The Higgs potential for the $SO(3)_F$ Higgs triplets has the following general form before symmetry breaking

$$\begin{aligned}V_\varphi &= \frac{1}{2}\mu^2(\varphi^\dagger\varphi) + \frac{1}{2}\mu^{'2}(\varphi'^\dagger\varphi') + \frac{1}{2}\mu^{''2}(\varphi''^\dagger\varphi'') \\ &+\frac{1}{4}\lambda(\varphi^\dagger\varphi)^2 + \frac{1}{4}\lambda'(\varphi'^\dagger\varphi')^2 + \frac{1}{4}\lambda''(\varphi''^\dagger\varphi'')^2 \\ &+\frac{1}{2}\kappa_1(\varphi^\dagger\varphi)(\varphi'^\dagger\varphi') + \frac{1}{2}\kappa_1'(\varphi^\dagger\varphi)(\varphi''^\dagger\varphi'') \\ &+\frac{1}{2}\kappa_1''(\varphi'^\dagger\varphi')(\varphi''^\dagger\varphi'') + \frac{1}{2}\kappa_2(\varphi^\dagger\varphi')(\varphi'^\dagger\varphi)\end{aligned}$$

$$+\frac{1}{2}\kappa_2'(\varphi^\dagger\varphi'')(\varphi''^\dagger\varphi)+\frac{1}{2}\kappa_2''(\varphi'^\dagger\varphi'')(\varphi''^\dagger\varphi') \ .$$

As the $SO(3)_F$ flavor symmetry is treated to be a gauge symmetry, one can always express the complex $SO(3)_F$ Higgs triplet field in terms of three rotational fields $\eta_i(x)$ and three amplitude fields $\rho_i(x)$

$$\begin{pmatrix}\varphi_1(x)\\ \varphi_2(x)\\ \varphi_3(x)\end{pmatrix} = e^{i\eta_i(x)t^i}\frac{1}{\sqrt{2}}\begin{pmatrix}\rho_1(x)\\ i\rho_2(x)\\ \rho_3(x)\end{pmatrix} \qquad (2)$$

Similar forms are for $\varphi'(x)$ and $\varphi''(x)$. Assuming that only the amplitude fields get VEVs after spontaneous symmetry breaking, namely $<\rho_i(x)>=\sigma_i$, $<\rho_i'(x)>=\sigma_i'$ and $<\rho_i''(x)>=\sigma_i''$, we then obtain the following equations from minimizing the Higgs potential[5]

$$\begin{aligned}&\sigma_1'=\sqrt{\xi}\sigma_1,\ \sigma_2'=\sqrt{\xi}\sigma_2,\ \sigma_3'=-\sqrt{\xi}\sigma_3,\\ &\sigma_1''=\sqrt{2\xi'}\sigma_1,\ \ \sigma_2''=-\sqrt{2\xi'}\sigma_2,\ \ \sigma_3''=0,\\ &\sigma_3^2=\sigma_1^2+\sigma_2^2=2\sigma_1^2=\sigma^2/2 \ . \qquad (3)\end{aligned}$$

where we have asummed a global minimum potential energy $V_\varphi|_{min}$ for varying ξ and ξ' at the minimizing point

$$\begin{aligned}V_\varphi|_{min}=&-\sigma^4(\lambda+\lambda'\xi^2+\lambda''\xi^{'2}\\ &+2\kappa_1\xi+2\kappa_1'\xi'+2\kappa_1''\xi\xi')/4 \qquad (4)\end{aligned}$$

with $\sigma^2=\sigma_1^2+\sigma_2^2+\sigma_3^2$, $\xi=\sigma^2/\sigma^{'2}$ and $\xi'=\sigma^2/\sigma^{''2}$. It is seen that with these considerations the VEVs are completely determined by the Higgs potential.

It is remarkable that with these relations the mass matrices of the neutrinos and charged leptons are simply given by

$$M_e=\frac{m_\tau}{2}\begin{pmatrix}\frac{1}{2} & \frac{1}{2}i & \frac{1}{\sqrt{2}}\\ \frac{1}{2}i & -\frac{1}{2} & \frac{1}{\sqrt{2}}i\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i & 1\end{pmatrix} \qquad (5)$$

$$+\frac{m_\mu}{2}\begin{pmatrix}\frac{1}{2} & \frac{1}{2}i & -\frac{1}{\sqrt{2}}\\ \frac{1}{2}i & -\frac{1}{2} & -\frac{1}{\sqrt{2}}i\\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i & 1\end{pmatrix}-\frac{m_e}{2}\begin{pmatrix}1 & i & 0\\ i & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$M_\nu=\hat{m}_\nu\begin{pmatrix}1 & 0 & \frac{1}{\sqrt{2}}\hat{\delta}_-\\ 0 & 1 & 0\\ \frac{1}{\sqrt{2}}\hat{\delta}_- & 0 & 1+\hat{\Delta}_-\end{pmatrix} \qquad (6)$$

3 Nearly Bimaximal Mixing

It is more remarkable that the mass matrix M_e can be diagonalized by a unitary bi-maximal mixing matrix U_e via $D_e = U_e^\dagger M_e U_e^*$ with

$$U_e^\dagger=\begin{pmatrix}\frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{2} & -\frac{1}{2}i & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & -\frac{1}{2}i & \frac{1}{\sqrt{2}}\end{pmatrix} \qquad (7)$$

and $D_e = diag.(m_e, m_\mu, m_\tau)$. The neutrino mass matrix can be easily diagonalized by an orthogonal matrix O_ν via $O_\nu^T M_\nu O_\nu$ with $(O_\nu)_{13}=\sin\theta_\nu\equiv s_\nu$ and $\tan 2\theta_\nu = \sqrt{2}\hat{\delta}_-/\hat{\Delta}_-$. Thus the CKM-type lepton mixing matrix U_{LEP} that appears in the interaction term $\mathcal{L}_W=\bar{e}_L\gamma^\mu U_{LEP}\nu_L W_\mu^- + H.c.$ is given by $U_{LEP}=U_e^\dagger O_\nu$. Explicitly, one has

$$U_{LEP}=\begin{pmatrix}\frac{1}{\sqrt{2}}ic_\nu & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}is_\nu\\ \frac{1}{2}c_\nu+\frac{1}{\sqrt{2}}s_\nu & -\frac{1}{2}i & \frac{1}{2}s_\nu-\frac{1}{\sqrt{2}}c_\nu\\ \frac{1}{2}c_\nu-\frac{1}{\sqrt{2}}s_\nu & -\frac{1}{2}i & \frac{1}{2}s_\nu+\frac{1}{\sqrt{2}}c_\nu\end{pmatrix}. \qquad (8)$$

The three neutrino masses are found to be

$$\begin{aligned}m_{\nu_e}&=\hat{m}_\nu[1-(\sqrt{\hat{\Delta}_-^2+2\hat{\delta}_-^2}-\hat{\Delta}_-)/2\ ]\\ m_{\nu_\mu}&=\hat{m}_\nu \qquad (9)\\ m_{\nu_\tau}&=\hat{m}_\nu[1+\hat{\Delta}_-+(\sqrt{\hat{\Delta}_-^2+2\hat{\delta}_-^2}-\hat{\Delta}_-)/2\ ].\end{aligned}$$

The similarity between the Higgs triplets $\varphi(x)$ and $\varphi'(x)$ naturally motivates us to consider an approximate (and softly broken) permutation symmetry between them. This implies that $|\hat{\delta}_-|<<1$. To a good approximation, the mass-squared differences are given by $\Delta m_{\mu e}^2\equiv m_{\nu_\mu}^2-m_{\nu_e}^2\simeq\hat{m}_\nu^2\hat{\Delta}_-(\hat{\delta}_-/\hat{\Delta}_-)^2$ and $\Delta m_{\tau\mu}^2\equiv m_{\nu_\tau}^2-m_{\nu_\mu}^2\simeq\hat{m}_\nu^2\hat{\Delta}_-(2+\hat{\Delta}_-)$, which leads to the approximate relation

$$\frac{\Delta m_{\mu e}^2}{\Delta m_{\tau\mu}^2}\simeq\left(\frac{\hat{\delta}_-}{\sqrt{2}\hat{\Delta}_-}\right)^2\simeq s_\nu^2=2|U_{e3}|^2<<1$$

$$\simeq\begin{array}{ll}0.2\sim 0.09 & MSW-LMA\\ 0.02\sim 0.002 & MSW-LOW\\ 10^{-7} & Vacuum\ Oscillation\end{array} \qquad (10)$$

which implies that once the mixing element $|U_{e3}|$ is determined, such a relation will tell us which solution should be favored.

When going back to the weak gauge and charged-lepton mass basis, the neutrino mass matrix gets the following interesting form

$$M_\nu/\hat{m}_\nu \simeq \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}}i & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}}i & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \frac{\hat{\delta}_-}{2}\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}}i & -1 & 0 \\ \frac{1}{\sqrt{2}}i & 0 & 1 \end{pmatrix} + \frac{\hat{\Delta}_-}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \tag{11}$$

As $(M_\nu)_{ee} = 0$, the neutrinoless double beta decay is forbiden in the model. Thus the neutrino masses can be approximately degenerate and large enough ($\hat{m}_\nu = O(1)$ eV) to play a significant cosmological role. Note that our scenario was shown to remain stable after considering renormalization group effects[5].

4 Lepton Flavor Violations

The mass matrix of gauge fields A_μ^i is

$$M_F^2 = \frac{m_F^2}{3}\begin{pmatrix} 2(\xi_+ + \xi') & 0 & -\sqrt{2}\xi_- \\ 0 & 3\xi_+ + \xi' & 0 \\ -\sqrt{2}\xi_- & 0 & 3\xi_+ + \xi' \end{pmatrix} \tag{12}$$

with $m_F^2 = 3g_3^{'2}\sigma^2/8$ and $\xi_\pm = (1 \pm \xi)/2$. This mass matrix is diagonalized by an orthogonal matrix O_F via $O_F^T M_F^2 O_F$ with $(O_F)_{13} = \sin\theta_F \equiv s_F$ and $\tan 2\theta_F = 2\sqrt{2}\xi_-/(\xi_+ - \xi')$. Denoting the physical gauge fields as F_μ^i, we then have $A_\mu^i = O_F^{ij} F_\mu^j$. In the physical mass basis, we have for gauge interactions

$$\mathcal{L}_F = \frac{g_3'}{2} F_\mu^i \bar{\nu}_L t^j O_F^{ji} \gamma^\mu \nu_L + \frac{g_3'}{2} F_\mu^i \left(\bar{e}_L V_e^i \gamma^\mu e_L - \bar{e}_R V_e^{i*} \gamma^\mu e_R\right) \tag{13}$$

with $V_e^i = U_e^\dagger t^j U_e O_F^{ji}$. Explicitly, we find

$$V_e^1 = \begin{pmatrix} c_F & i\frac{1}{2}s_F & -i\frac{1}{2}s_F \\ -i\frac{1}{2}s_F & \frac{1}{2}c_F + \frac{1}{\sqrt{2}}s_F & \frac{1}{2}c_F \\ i\frac{1}{2}s_F & \frac{1}{2}c_F & \frac{1}{2}c_F - \frac{1}{\sqrt{2}}s_F \end{pmatrix}$$

$$V_e^2 = \begin{pmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & 0 & i\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -i\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \tag{14}$$

$$V_e^3 = \begin{pmatrix} -s_F & i\frac{1}{2}c_F & -i\frac{1}{2}c_F \\ -i\frac{1}{2}c_F & -\frac{1}{2}s_F + \frac{1}{\sqrt{2}}c_F & -\frac{1}{2}s_F \\ i\frac{1}{2}c_F & -\frac{1}{2}s_F & -\frac{1}{2}s_F - \frac{1}{\sqrt{2}}c_F \end{pmatrix}.$$

Thus the $SO(3)_F$ gauge interactions allow lepton flavor violating process $\mu \to 3e$, its branch ratio is

$$Br(\mu \to 3e) = \left(\frac{v}{\sigma}\right)^4 \frac{2\xi_-^2}{[(3\xi_+ + \xi')(\xi_+ + \xi') - \xi_-^2]^2}. \tag{15}$$

with $v = 246$GeV. For $\sigma \sim 10^3 v$, the branch ratio could be very close to the present experimental upper bound $Br(\mu \to 3e) < 1 \times 10^{-12}$. Thus when taking the mixing angle θ_F and the coupling constant g_3' for the $SO(3)_F$ gauge bosons to be at the same order of magnitude as those for the electroweak gauge bosons, we find that masses of the $\mathrm{SO}(3)_F$ gauge bosons are at the order of magnitudes $m_{F_i} \sim 10^3 m_W \simeq 80$ TeV. For smaller mising angle θ_F, the $\mathrm{SO}(3)_F$ gauge boson masses m_{F_i} could be below 1 TeV.

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